

General Covariance in Quantum Gravity

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Abstract

The question of general covariance in quantum gravity is considered in the first post-Newtonian approximation. Transformation properties of observable quantities under deformations of a reference frame, induced by variations of the gauge conditions fixing general invariance, are determined. It is found that the one-loop contributions violate the principle of general covariance, in the sense that the quantities which are classically invariant under such deformations take generally different values in different reference frames. The relative value of this violation is of the order $1/N$, where N is the number of particles in a gravitating body.

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I. INTRODUCTION

Self-consistent quantization of gravitation is usually considered as a high-energy problem. There is a number of models of quantum gravity none of which has succeeded in reconciliation of renormalizability with basic principles of the scattering theory, such as unitarity and causality. It is important, however, that the low-energy properties of quantum gravity are universal whatever the ultimate theory, in that they are determined solely by the lowest order Einstein theory. Investigation of the low-energy limit allows one to draw important conclusions about the synthesis of quantum theory and gravitation.

One of the main principles underlying Einstein's general relativity is the principle of general covariance. It states that arbitrary coordinate transformations must leave the form of dynamical equations unchanged. Since quantum theory deals with fields rather than coordinate transformations, another formulation of this principle is appropriate for the purposes of quantization: dynamical equations must be invariant under arbitrary spacetime diffeomorphisms. This formulation reveals general relativity as a gauge theory. From this point of view, any specific choice of coordinate system, or more precisely, of reference frame, is equivalent to imposition of an appropriate set of gauge conditions on the metric field, a change in the gauge conditions being equivalent to a spacetime diffeomorphism.

In classical theory, there is no difference between the two points of view. General covariance of the theory implies that it is diffeomorphism-invariant, and vice versa. An important difference appears, however, in quantum theory. While the notion of reference frame retains its essentially classical content, components of the metric are promoted into operators, and so are generators of the gauge transformations. Furthermore, the gauge conditions become operator relations. Used in the classical theory as a means for defining reference frames and their transformations, these notions thus lose their direct interpretation in quantum domain. In this respect, a natural question arises about relevant interpretation of the above-mentioned operator relations, and their role in defining reference frames in quantum gravity.

In connection with the above statement of the problem the following circumstance should be emphasized. It is widely believed that the characteristic length scale where quantum gravity effects come into play is given by the Planck length

$$l_P = \sqrt{\frac{G\hbar}{c^3}}. \quad (1)$$

The quantum gravitational corrections to the classical laws are thus expected to be of the relative order $l_P^2/l^2 = O(\hbar)$, where l is the characteristic length of the problem under consideration. Since these corrections reflect quantum properties of the spacetime itself, one might doubt relevance of the notion of coordinate system in the classical sense outlined above. One should note, however, that this reasoning is based on the assumption that the Planck length is the *only* scale of quantum gravitational effects. As far as pure gravity is considered, this assumption is certainly true since l_P^2/\hbar is the only dimensional constant entering the Einstein action

$$S = -\frac{c^3}{16\pi G} \int d^4x \sqrt{-g} R. \quad (2)$$

In the presence of a matter field, however, another length scale appears – the gravitational radius

$$r_g = \frac{2Gm}{c^2}, \quad (3)$$

where m is the mass of the field quanta. As a matter of fact, along with terms proportional to \hbar , the radiative gravitational corrections also contain terms independent of the Planck constant [1]¹. Thus, the question whether the quantum gravity effects appear already at the order \hbar^0 , or not, is the question of *correspondence* between classical and quantum theories.

The problem of establishing the correct correspondence in quantum gravity was considered in detail in Refs. [2,3]. It was shown, in particular, that the approach using the S-matrix potential is inadequate for this purpose, and the correct correspondence between classical and quantum theories is to be established in terms of the effective (mean) fields, rather than the S-matrix. This suggestion is underlined by an observation that the n -loop radiative contribution to the n th post-Newtonian correction to the mean gravitational field of a body with mass M , consisting of $N = M/m$ elementary particles with mass m , contains an extra factor of $1/N^n$ in comparison with the corresponding tree contribution. Thus, the effective gravitational field produced by the body turns into the classical solution of the Einstein equations in the limit $N \rightarrow \infty$ (and therefore, $M \rightarrow \infty$). It was also shown in Ref. [2] that the S-matrix gravitational potential becomes Newtonian in the same limit, hence, it fails to describe whatever non-Newtonian interactions of *macroscopic* bodies.

An immediate consequence of this interpretation is that in the case of *finite* N , the loop corrections of the order \hbar^0 describe deviations of the spacetime metric from classical solutions of the Einstein equations, implying that (3) is the true scale of quantum gravity effects as $\hbar \rightarrow 0$.

We can now reformulate the initial problem more precisely as follows. On the one hand, the \hbar^0 radiative corrections are of the same functional form as the post-Newtonian corrections predicted by classical general relativity, and therefore must be treated on an equal footing with the latter. On the other hand, their transformation under transitions between different gauge conditions is expected to be more complicated. The question is what the law of this transformation and its physical interpretation are. This is actually the question of *general covariance in quantum gravity*.²

The aim of the present paper is to investigate this problem at the first post-Newtonian approximation. The general approach used for this purpose is outlined in Sec. II, where also the formulation of Einstein's general covariance at the classical level is given in terms of quantum field theory. The transformation law of the effective metric under variations of the Feynman parameter (more generally, matrix of parameters) weighting the gauge conditions in the action is established in Sec. III A. It is shown that these variations induce spacetime diffeomorphisms, and hence do not change the values of observables. Comparison of this result with classical theory is made. Transformation properties of the effective metric under

¹This property is specifically gravitational, connected with the fact that the strength of this interaction is determined by the masses of particles.

²It is important that the restriction to zeroth order in the Planck constant allows one to avoid, at least formally, the difficult question about general physical interpretation of the quantum corrections of higher orders.

variations of the gauge conditions themselves are investigated in Sec. IIIB. This is done using the simplest model of the scalar field minimally coupled to the gravitational field. The obtained results are discussed in Sec. IV.

Condensed notations of DeWitt [4] are in force throughout this paper. Also, right and left derivatives with respect to the fields and the sources, respectively, are used. The dimensional regularization of all divergent quantities is assumed.

II. PRELIMINARIES.

Before going into detailed discussion of the question of general covariance in quantum gravity, let us describe the general setting we will be working in.

A. Frame of reference and interacting fields.

First of all, we should set a frame of reference, *i.e.*, a system of idealized reference bodies with respect to which the 4-position in spacetime can be fixed. Let us assume, for definiteness, that the frame of reference is realized by means of an appropriate distribution of electrically charged matter. For simplicity, the energy-momentum of matter, as well as of the electromagnetic field it produces, will be assumed sufficiently small so as not to alter the gravitational field under consideration. The 4-position in spacetime can be determined by exchanging electromagnetic signals with a number of charged matter species. The electric charge distributions σ_a of the latter are thus supposed to be in a one-to-one correspondence with the spacetime coordinates x_μ ,

$$\sigma_a \leftrightarrow x_\mu ,$$

where index a enumerates the species. The $\sigma_a(x)$ will be assumed smooth scalar functions. Physical properties of the reference frame are determined by the action S_σ which specific form is of no importance for us.

Next, let us consider a system of interacting gravitational and matter fields. The latter are arbitrary species, bosons or fermions, self-interacting or not, denoted collectively by ϕ_i , $i = 1, 2, \dots, k$. Dynamical variables of the gravitational field are $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. Dynamics of the system is described by the action $S + S_\phi$, where S_ϕ is the matter action, and S is given³ by Eq. (2).

The total action $S + S_\phi + S_\sigma$ is invariant under the gauge transformations

$$\delta h_{\mu\nu} = \xi^\alpha \partial_\alpha h_{\mu\nu} + (\eta_{\mu\alpha} + h_{\mu\alpha}) \partial_\nu \xi^\alpha + (\eta_{\nu\alpha} + h_{\nu\alpha}) \partial_\mu \xi^\alpha \equiv D_{\mu\nu}^\alpha \xi_\alpha , \quad (4)$$

$$\delta \phi_i = D_i^\alpha \xi_\alpha , \quad (5)$$

$$\delta \sigma_a = \sigma_{a,\alpha} \xi^\alpha \quad (6)$$

The generators $D_{\mu\nu}, D_i$ span the closed algebra

³Our notation is $R_{\mu\nu} \equiv R_{\mu\alpha\nu}^\alpha = \partial_\alpha \Gamma_{\mu\nu}^\alpha - \dots$, $R \equiv R_{\mu\nu} g^{\mu\nu}$, $g \equiv \det g_{\mu\nu}$, $g_{\mu\nu} = \text{sgn}(+, -, -, -)$, $\eta_{\mu\nu} = \text{diag}\{+1, -1, -1, -1\}$. The Minkowski tensor η is used to raise and lower tensor indices. The units in which $\hbar = c = 16\pi G = 1$ are chosen in what follows.

$$\begin{aligned}
D_{\mu\nu}^{\alpha,\sigma\lambda} D_{\sigma\lambda}^\beta - D_{\mu\nu}^{\beta,\sigma\lambda} D_{\sigma\lambda}^\alpha &= f^{\alpha\beta}{}_\gamma D_{\mu\nu}^\gamma, \\
D_i^{\alpha,k} D_k^\beta - D_i^{\beta,k} D_k^\alpha &= f^{\alpha\beta}{}_\gamma D_i^\gamma,
\end{aligned} \tag{7}$$

where the "structure constants" $f^{\alpha\beta}{}_\gamma$ are defined by

$$f^{\alpha\beta}{}_\gamma \xi_\alpha \eta_\beta = \xi_\alpha \partial^\alpha \eta_\gamma - \eta_\alpha \partial^\alpha \xi_\gamma. \tag{8}$$

Let the gauge-fixing action be written in the form

$$S_{\text{gf}} = \left(F_\alpha - \frac{1}{2} \pi^\beta \zeta_{\beta\alpha} \right) \pi^\alpha, \tag{9}$$

where F_α is a set of functions of the fields $h_{\mu\nu}$, fixing general invariance, π^α auxiliary fields introducing the gauge, and $\zeta_{\alpha\beta}$ a non-degenerate matrix weighting the functions F_α ; the particular choice $\zeta_{\alpha\beta} = \xi \eta_{\alpha\beta}$ corresponds to the well-known Feynman weighting of the gauge conditions. Introducing the ghost fields c_α , \bar{c}^α , we write the Faddeev-Popov action [5]

$$S_{FP} = S + S_\phi + S_\sigma + S_{\text{gf}} + \bar{c}^\beta F_\beta^{\mu\nu} D_{\mu\nu}^\alpha c_\alpha. \tag{10}$$

S_{FP} is invariant under the following Becchi-Rouet-Stora-Tyutin (BRST) transformations [6]

$$\begin{aligned}
\delta h_{\mu\nu} &= D_{\mu\nu}^\alpha c_\alpha \lambda, \\
\delta \phi_i &= D_i^\alpha c_\alpha \lambda, \\
\delta \sigma_a &= \sigma_{a,\alpha} c^\alpha \lambda, \\
\delta c_\gamma &= -\frac{1}{2} f^{\alpha\beta}{}_\gamma c_\alpha c_\beta \lambda, \\
\delta \bar{c}^\alpha &= \pi^\alpha \lambda, \\
\delta \pi^\alpha &= 0,
\end{aligned}$$

where λ is a constant anticommuting parameter.

B. Generating functionals and Slavnov identities.

The generating functional of Green functions has the form

$$Z[J, K] = \int d\Phi \exp\{i(\Sigma + \bar{\beta}^\alpha c_\alpha + \beta_\alpha \bar{c}^\alpha + t^{\mu\nu} h_{\mu\nu} + j^i \phi_i + s^a \sigma_a)\}, \tag{11}$$

where

$$\Sigma = S_{FP} + k^{\mu\nu} D_{\mu\nu}^\alpha c_\alpha + q^i D_i^\alpha c_\alpha + r^a \sigma_{a,\alpha} c^\alpha - \frac{l^\gamma}{2} f^{\alpha\beta}{}_\gamma c_\alpha c_\beta + n_\alpha \pi^\alpha,$$

$\{t, j, s, \bar{\beta}, \beta\} \equiv J$ ordinary sources, and $\{k, q, r, l, n\} \equiv K$ the BRST-transformation sources [7] for the fields $\{h, \phi, \sigma, c, \bar{c}\} \equiv \Phi$, respectively.

The functional Σ can be written [8]

$$\Sigma(\Phi, K) = \Sigma^r \left(\Phi, \frac{\delta \Psi(\Phi, K)}{\delta \Phi} \right),$$

where the reduced action

$$\Sigma^r(\Phi, K) = S + S_\phi + S_\sigma + k^{\mu\nu} D_{\mu\nu}^\alpha c_\alpha + q^i D_i^\alpha c_\alpha + r^a \sigma_{a,\alpha} c^\alpha - \frac{l^\gamma}{2} f^{\alpha\beta}_\gamma c_\alpha c_\beta + n_\alpha \pi^\alpha,$$

and the gauge fermion

$$\Psi(\Phi, K) = K\Phi + \bar{c}^\alpha \left(F_\alpha - \frac{1}{2} \pi^\beta \zeta_{\beta\alpha} \right).$$

Hence, under infinitesimal variation of the gauge conditions, Σ transforms as

$$\delta\Sigma(\Phi, K) = \frac{\delta\Delta\Psi(\Phi, K)}{\delta\Phi} \frac{\delta\Sigma(\Phi, K)}{\delta K}.$$

The corresponding variation of the generating functional

$$\delta Z[J, K] = i \int d\Phi \exp\{i(\Sigma + J\Phi)\} \frac{\delta\Delta\Psi(\Phi, K)}{\delta\Phi} \frac{\delta\Sigma(\Phi, K)}{\delta K}.$$

Integrating by parts and omitting $\delta^2\Sigma/\delta K\delta\Phi \sim \delta(0)$ in the latter equation gives

$$\delta Z[J, K] = iJ \frac{\delta}{\delta K} \int d\Phi \Delta\Psi \exp\{i(\Sigma + J\Phi)\}. \quad (12)$$

Since Σ is invariant under the BRST transformation (11), a BRST change of integration variables in Eq. (11) gives the Slavnov identity for the generating functional

$$J \frac{\delta Z}{\delta K} = 0, \quad (13)$$

which allows one to rewrite Eq. (12) in terms of the generating functional of connected Green functions, $W = -i \ln Z$,

$$\delta W[J, K] = J \frac{\delta}{\delta K} \langle \Delta\Psi \rangle, \quad (14)$$

where $\langle X \rangle$ denotes the functional averaging of X [9].

C. General covariance at the tree level.

From the point of view of the general formalism outlined in the preceding sections, the classical Einstein theory corresponds to the tree approximation of the full quantum theory. The tree contributions to the expectation values of field operators coincide with the corresponding classical fields, and the effective equations of motion

$$\left\langle \frac{\delta\Sigma}{\delta h_{\mu\nu}} \right\rangle + t^{\mu\nu} = 0,$$

expressing the translation invariance of the functional integral measure, go over into the classical Einstein equations. The results of the preceding section allow one, in particular, to reestablish the general covariance of these equations.

As was mentioned in the Introduction, coordinate transformations are replaced in the quantum theory by the field transformations. In particular, transformations of the reference frame are represented by variations of the fields σ_i , induced by appropriate variations of the gauge conditions. Namely, it follows from Eq. (14) at the tree level that a gauge variation ΔF_α induces the following variations of the metric and reference fields

$$\delta g_{\mu\nu} = g_{\mu\nu,\alpha}\Xi^\alpha + g_{\mu\alpha,\nu}\Xi^\alpha + g_{\nu\alpha,\mu}\Xi^\alpha, \quad (15)$$

$$\delta\sigma_a = \sigma_{a,\alpha}\Xi^\alpha, \quad \Xi_\alpha = \langle c_\alpha \Delta\Psi \rangle, \quad (16)$$

where $\Delta\Psi$ is the corresponding variation of the gauge fermion.

The functions $g_{\mu\nu}, \sigma_a$ undergo the same variations (15), (16) under the spacetime diffeomorphism

$$x^\mu \rightarrow x^\mu + \delta x^\mu,$$

with $\delta x^\mu = -\Xi^\mu$. Let us consider any quantity entering the Einstein equations, for instance, the scalar curvature R . Under the above change of gauge conditions, the tree value of R measured at a point σ^0 of the reference frame remains unchanged,

$$\delta R[g(x(\sigma^0))] = \frac{\delta R}{\delta g_{\mu\nu}}\delta g_{\mu\nu} + \frac{\partial R}{\partial x^\mu}\delta x^\mu = \frac{\partial R}{\partial x^\alpha}\Xi^\alpha - \frac{\partial R}{\partial x^\mu}\Xi^\mu = 0. \quad (17)$$

Analogously, the tree contribution to any tensor quantity $O_{\mu\nu\dots}$ (or $O^{\mu\nu\dots}$), for instance, the metric $g_{\mu\nu}$ itself, calculated at a fixed reference point σ^0 , transforms covariantly (contravariantly), as prescribed by the position of the tensor indices of the corresponding operator. This is the manifestation of general covariance of the classical Einstein theory in terms of quantum field theory.

III. GENERAL COVARIANCE AT THE ONE-LOOP ORDER.

Let us now consider the transformation properties of the one-loop contributions. As in Sec. IIC, we have to determine the effect of an arbitrary gauge variation on the value of the effective metric, and also on the functions σ_a , *i.e.*, on the structure of the reference frame. After that, the transformation law of observables, defined generally as the diffeomorphism-invariant functions of the metric and reference fields, can be determined in the way followed in Sec. IIC. For definiteness, we will deal below with the scalar curvature R . Since we are interested in the one-loop contribution to the first post-Newtonian correction, we can linearize R in $h_{\mu\nu}$:

$$R = \partial^\mu \partial^\nu h_{\mu\nu} - \square h, \quad h \equiv \eta^{\mu\nu} h_{\mu\nu}. \quad (18)$$

The transformation properties of R under variations of the weighting matrix $\zeta_{\alpha\beta}$ are considered in Sec. III A, and under variations of the gauge functions F_α themselves in Sec. III B.

A. Dependence of observables on weighting parameters.

Dependence of the effective fields on the weighting matrix $\zeta_{\alpha\beta}$ can be determined in a quite general way without specifying neither the gauge functions F_α , nor the properties of gravitating matter fields. We begin with the classical theory in Sec. III A 1, and then consider the one-loop order in Sec. III A 2.

1. The tree level.

As we saw in Sec. IIC, arbitrary gauge variations lead to the transformations of the classical fields, equivalent to the spacetime diffeomorphisms, and thus do not affect the values of R . In the particular case of variations of the weighting matrix, however, not only R , but also the effective fields themselves remain unchanged. This means that the structure of reference frames in classical theory is determined by the functions F_α only. As this differs in the full quantum theory, a somewhat more detailed discussion of this issue will be given in this Section.

To show the $\zeta_{\alpha\beta}$ -independence of the classical metric, let us first integrate the auxiliary fields π^α out of the gauge-fixing action (9),

$$S_{\text{gf}} \rightarrow S_{\text{gf}}^\xi = \frac{1}{2} F_\alpha \xi^{\alpha\beta} F_\beta, \quad \zeta_{\alpha\beta} \xi^{\beta\gamma} = \delta_\alpha^\gamma. \quad (19)$$

The classical equations of motion thus become

$$\frac{\delta(S + S_{\text{gf}}^\xi)}{\delta g_{\mu\nu}} = -T^{\mu\nu}, \quad (20)$$

where $T^{\mu\nu}$ is the energy-momentum tensor of matter. Using invariance of the action S under the gauge transformations

$$\delta g_{\mu\nu} = \xi^\alpha \partial_\alpha g_{\mu\nu} + g_{\mu\alpha} \partial_\nu \xi^\alpha + g_{\nu\alpha} \partial_\mu \xi^\alpha \equiv \nabla_\mu \xi_\nu, \quad (21)$$

and taking into account the “conservation law” $\nabla_\mu T^{\mu\nu} = 0$, one has from Eq. (20)

$$F_\alpha \xi^{\alpha\beta} \frac{\delta F_\beta}{\delta g_{\mu\nu}(x)} \nabla_\mu^x \delta(x - y) = 0. \quad (22)$$

The matrix $M_\beta^\nu(x, y) = \delta F_\beta / \delta g_{\mu\nu}(x) \nabla_\mu^x \delta(x - y)$ is non-degenerate; its determinant $\Delta \equiv \det M_\beta^\nu(x, y)$ is just the Faddeev-Popov determinant, and therefore $\Delta \neq 0$. Hence, one has from Eq. (22) $F_\alpha \xi^{\alpha\beta} = 0$, and, in view of non-degeneracy of $\xi^{\alpha\beta}$, $F_\alpha = 0$. The classical metric is thus independent of the choice of the matrix $\xi^{\alpha\beta}$, and in particular, of the replacements $F_\alpha \rightarrow A_\alpha^\beta F_\beta$. One can put this in another way by saying that the weighting matrix has no geometrical meaning in classical theory.

This differs, however, in quantum domain. The classical equations (20) are replaced in quantum theory by the effective equations

$$\frac{\delta \Gamma}{\delta g_{\mu\nu}^{\text{eff}}} = -T_{\text{eff}}^{\mu\nu},$$

where Γ , $g_{\mu\nu}^{\text{eff}}$, and $T_{\text{eff}}^{\mu\nu}$ are the effective action, metric, and energy-momentum tensor of matter, respectively. In general, the fields $g_{\mu\nu}^{\text{eff}}$ do not satisfy the gauge conditions $F_\alpha = 0$, and moreover, depend on the choice of the weighting matrix $\xi^{\alpha\beta}$; $\xi^{\alpha\beta}$ -independence is inherited only by the tree contribution.

Dependence on the choice of the weighting matrix generally represents an excess of the gauge arbitrariness over the arbitrariness in the choice of reference frame; it is therefore

a potential source of ambiguity in the values of observables. This dependence causes no gauge ambiguity of observables only if it reduces to the symmetry transformations. In other words, under (infinitesimal) variations of the matrix $\xi^{\alpha\beta}$, the fields $g_{\mu\nu}^{\text{eff}}$ must transform as in Eq. (21)

$$\delta g_{\mu\nu}^{\text{eff}} = \Xi^\alpha \partial_\alpha g_{\mu\nu}^{\text{eff}} + g_{\mu\alpha}^{\text{eff}} \partial_\nu \Xi^\alpha + g_{\nu\alpha}^{\text{eff}} \partial_\mu \Xi^\alpha, \quad (23)$$

with some functions Ξ^α . It will be shown in the following Section that this is the case indeed.

2. The one-loop level.

Let us now turn to examination of the gauge dependence of \hbar^0 loop contribution to the effective gravitational field. This contribution comes from diagrams in which virtual propagation of matter fields is near their mass shells, and is represented by terms containing the root singularity with respect to the momentum transfer between gravitational and matter fields. In the first post-Newtonian approximation, the only diagram we need to consider is the one-loop diagram pictured in Fig. 1. As a simple analysis shows, other one-loop diagrams do not contain the root singularities, while the higher-loop diagrams are of higher orders in the Newton constant.

According to Eq. (14), under a variation $\Delta\Psi$ of the gauge fermion, variation of the effective gravitational field

$$h_{\mu\nu}^{\text{eff}} = \frac{\delta W}{\delta t^{\mu\nu}}$$

has the form

$$\delta h_{\mu\nu}^{\text{eff}} = \left(\frac{\delta}{\delta k^{\mu\nu}} \langle \Delta\Psi \rangle + j^i \frac{\delta^2}{\delta t^{\mu\nu} \delta q^i} \langle \Delta\Psi \rangle \right)_{\substack{J \setminus j=0 \\ K=0}}, \quad (24)$$

where $J \setminus j$ means that the source j is excluded from J . We are interested presently in variations of the weighting matrix $\xi^{\mu\nu}$, therefore,

$$\Delta\Psi(\Phi, K) = -\frac{\bar{c}^\alpha}{2} \pi^\beta \Delta\zeta_{\beta\alpha},$$

or, integrating π^α out,

$$\Delta\Psi(\Phi, K) = \frac{\bar{c}^\alpha}{2} \zeta_{\alpha\beta} \Delta\xi^{\beta\gamma} F_\gamma. \quad (25)$$

According to general rules, in order to find the contribution of a diagram with n external ϕ -lines, one has to take the n th derivative of the right hand side of Eq. (24) with respect to j^i , multiply the result by the product of n factors $e_i(q^2 - m^2)$, where e_i, q are the 4-momentum and polarization of the external ϕ -field quanta, and set $q^2 = m^2$ afterwards. The second term on the right of Eq. (24) is proportional to the source j^i contracted with the vertex $D_i^\alpha c_\alpha$. This term represents contribution of the graviton propagators ending on the external matter lines. Multiplied by $(q^2 - m^2)$, it gives rise to a non-zero value as $q^2 \rightarrow m^2$ only if the corresponding diagram is one-particle-reducible with respect to the ϕ -line, in which case it describes the variation of $h_{\mu\nu}^{\text{eff}}$ under the gauge variation of external matter

lines. It is well-known, however, that ϕ -operators must be renormalized⁴ so as to cancel all the radiative corrections to the external lines⁵. Therefore, this term can be omitted, and Eq. (24) rewritten finally as

$$\delta h_{\mu\nu}^{\text{eff}} = \frac{1}{2} \zeta_{\alpha\beta} \Delta^{\xi\beta\gamma} \left. \frac{\delta \langle \bar{c}^\alpha F_\gamma \rangle}{\delta k^{\mu\nu}} \right|_{\substack{J \setminus j=0 \\ K=0}}. \quad (26)$$

The one-loop diagrams contributing to the right hand side of Eq. (26), giving rise to the root singularity, are pictured in Fig. 2. Let us consider the diagram of Fig. 2(a) first. It turns out that this diagram is actually free of the root singularity despite the presence of the internal ϕ -line. The rightmost vertex in this diagram is generated by $\bar{c}^\alpha F_\gamma^{(1)}$, where $F_\gamma^{(1)}$ denotes the linear part of F_γ . The graviton propagator connecting this vertex to the ϕ -line can be expressed through the ghost propagator with the help of the equation

$$\xi^{\alpha\beta} F_\beta^{(1),\sigma\lambda} G_{\sigma\lambda\mu\nu} = D_{\mu\nu}^{(0)\beta} \tilde{G}_\beta^\alpha, \quad D_{\mu\nu}^{(0)\beta} \equiv D_{\mu\nu}^\beta \Big|_{h=0}, \quad (27)$$

where $G_{\mu\nu\sigma\lambda}$, \tilde{G}_β^α are the graviton and ghost propagators defined by

$$\left. \frac{\delta^2 (S + S_{\text{gf}}^\xi)}{\delta h_{\rho\tau} \delta h_{\mu\nu}} \right|_{h=0} G_{\mu\nu\sigma\lambda} = -\delta_{\sigma\lambda}^{\rho\tau}, \quad \delta_{\sigma\lambda}^{\rho\tau} = \frac{1}{2} (\delta_\sigma^\rho \delta_\lambda^\tau + \delta_\sigma^\tau \delta_\lambda^\rho),$$

and

$$F_\alpha^{(1),\mu\nu} D_{\mu\nu}^{(0)\beta} \tilde{G}_\beta^\gamma = -\delta_\alpha^\gamma,$$

respectively. Equation (27) is the Slavnov identity (13) at the tree level, differentiated twice with respect to $t^{\mu\nu}$, β_α . Using this identity in the diagram Fig. 2(a) we see that the ghost propagator is attached to the matter line through the generator $D_{\mu\nu}^{(0)\alpha}$. On the other hand, the action S_ϕ is invariant under the gauge transformations (4),

$$\frac{\delta S_\phi}{\delta \phi_i} D_i^\alpha + \frac{\delta S_\phi}{\delta h_{\mu\nu}} D_{\mu\nu}^\alpha = 0. \quad (28)$$

Differentiating this identity with respect to ϕ_k , setting $h_{\mu\nu} = 0$, and taking into account that the external ϕ -lines are on the mass shell

$$\left. \frac{\delta S_\phi^{(2)}}{\delta \phi_i} \right|_{h=0} = 0,$$

⁴One might think that the gauge dependence of the renormalization constants could spoil the above derivation of Eq. (14). In fact, this equation holds true for renormalized as well as unrenormalized quantities [9].

⁵The above discussion is nothing but the well-known reasoning underlying the proof of gauge-independence of the S -matrix [10].

where $S_\phi^{(2)}$ denotes the part of S_ϕ bilinear in ϕ , the $\phi^2 h$ vertex can be rewritten

$$\left. \frac{\delta^2 S_\phi^{(2)}}{\delta\phi_k \delta h_{\mu\nu}} \right|_{h=0} D_{\mu\nu}^{(0)\alpha} = - \left. \frac{\delta^2 S_\phi^{(2)}}{\delta\phi_i \delta\phi_k} \right|_{h=0} D_i^\alpha. \quad (29)$$

Thus, under contraction with the vertex factor, the ϕ -particle propagator, G_{ik}^ϕ , satisfying

$$\left. \frac{\delta^2 S_\phi^{(2)}}{\delta\phi_i \delta\phi_k} \right|_{h=0} G_{kl}^\phi = -\delta_l^i,$$

cancels out

$$G_{kl}^\phi \left. \frac{\delta^2 S_\phi^{(2)}}{\delta\phi_k \delta h_{\mu\nu}} \right|_{h=0} D_{\mu\nu}^{(0)\alpha} = D_l^\alpha. \quad (30)$$

We conclude that the \hbar^0 contribution of the diagram Fig. 2(a) is zero. As to the rest of diagrams, they are all proportional to the generator $D_{\mu\nu}^{(0)\alpha}$. Thus, the right hand side of Eq. (26) can be written

$$\delta h_{\mu\nu}^{\text{eff}} = D_{\mu\nu}^{(0)\alpha} \Xi_\alpha + O(\hbar), \quad \Xi_\alpha = \frac{1}{2} \tilde{G}_\alpha^\beta \zeta_{\beta\gamma} \Delta \xi^{\gamma\delta} \langle F_\delta \rangle.$$

Since Ξ_α are of the order G^2 , one can also write, within the accuracy of the first post-Newtonian approximation,

$$\delta h_{\mu\nu}^{\text{eff}} = D_{\mu\nu}^\alpha \Xi_\alpha, \quad (31)$$

where $D_{\mu\nu}^\alpha$ are defined by Eq. (4) with $h_{\mu\nu} \rightarrow h_{\mu\nu}^{\text{eff}}$.

We thus see that under variations of the weighting matrix, the effective metric does transform according to Eq. (23). To determine the effect of these variations on the values of observables, one has to find also the induced transformation of the reference frame, *i.e.*, of the functions σ_a . Obviously, the gauge variation of σ_a 's is represented by the same set of diagrams pictured in Fig. 2(b,c,d),⁶ with the only difference that the leftmost vertex $(\mu\nu)$ in these diagrams is now generated by $\sigma_{a,\alpha} c^\alpha$ instead of $D_{\mu\nu}^\alpha c_\alpha$. Thus, under variations of the weighting matrix, the functions $\sigma_a(x)$ transform according to

$$\delta \sigma_a = \sigma_{a,\alpha} \Xi^\alpha, \quad (32)$$

where Ξ^α 's are the same as in Eq. (31).

Equations (31) and (32) are of the same form as Eqs. (15) and (16), respectively, which implies that the value of any observable O is invariant under variations of the weighting matrix,

⁶The diagram of Fig. 2(a) does not contribute in this case.

$$\delta O[h^{\text{eff}}(x(\sigma^0))] = \frac{\delta O}{\delta h_{\mu\nu}^{\text{eff}}} \delta h_{\mu\nu}^{\text{eff}} + \frac{\partial O}{\partial x^\mu} \delta x^\mu = \frac{\partial O}{\partial x^\alpha} \Xi^\alpha - \frac{\partial O}{\partial x^\mu} \Xi^\mu = 0. \quad (33)$$

In particular,

$$\delta R[h^{\text{eff}}(x(\sigma^0))] = 0.$$

Furthermore, any tensor quantity $O_{\alpha\beta\dots}$ (or $O^{\mu\nu\dots}$), calculated at a fixed reference point σ^0 , transforms covariantly (contravariantly), as prescribed by the position of the tensor indices of the corresponding operator. This is in accord with the principle of general covariance.

B. Dependence of effective metric on the form of F_α 's.

Having established the general law of the effective metric transformation under variations of the weighting matrix, let us turn to investigation of the variations of the functions F_α themselves.

According to the general equation (24), a variation ΔF_α induces the following variation in the effective metric

$$\delta h_{\mu\nu}^{\text{eff}} = \frac{\delta \langle \bar{c}^\alpha \Delta F_\alpha \rangle}{\delta k^{\mu\nu}} \bigg|_{\substack{J \wedge j=0 \\ K=0}}. \quad (34)$$

The general structure of diagrams representing the one-loop contribution to the right hand side of this equation is the same as before and given by Fig. 2. Contribution of diagrams (b), (c), and (d) is again a spacetime diffeomorphism. In the present case, however, diagram (a) gives rise to a non-zero contribution already in the order \hbar^0 . Namely, it is not difficult to show that the combination $\Delta F_\alpha^{(1),\mu\nu} G_{\mu\nu\sigma\lambda}$ cannot be brought to the form proportional to the generator $D^{(0)}$. Note, first of all, that the variation of $\Delta F_\alpha^{(1),\mu\nu} G_{\mu\nu\sigma\lambda}$ with respect to $\xi^{\alpha\beta}$ is proportional to $D^{(0)}$; in the highly condensed DeWitt's notation,

$$\delta(\Delta F_1^{(1)} G) = \Delta F_1^{(1)} G(F_1^{(1)} \delta \xi F_1^{(1)}) G = \Delta F_1^{(1)} G F_1^{(1)} \delta \xi \zeta \tilde{G} D^{(0)},$$

where the Slavnov identity (27) has been used. Hence, without changing the \hbar^0 part of diagram (a), $\zeta_{\alpha\beta}$ can be set zero, in which case Eq. (27) gives $F_1^{(1)} G = 0$. Suppose that $\Delta F_\alpha^{(1),\mu\nu} G_{\mu\nu\sigma\lambda} = X_{\alpha\beta} D_{\sigma\lambda}^{(0)\beta}$, or shorter, $\Delta F_1^{(1)} G = X D^{(0)}$, with some X . Then one has $0 = \Delta F_1^{(1)} G F_1^{(1)} = X F_1^{(1)} D^{(0)} \equiv X M(h=0)$. Since the Faddeev-Popov determinant $\det M \neq 0$, it follows that $X = 0$. Thus, the argument used in the preceding section does not work, and the question is whether contribution of the diagram (a) can be actually represented in the form $D_{\mu\nu}^\alpha \Xi_\alpha$.

The answer to this question is negative, as an explicit calculation shows. This will be demonstrated below for the simplest case of scalar matter described by the action

$$S_\phi = \frac{1}{2} \int d^4x \sqrt{-g} \{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - m^2 \phi^2 \}, \quad (35)$$

and linear gauge conditions

$$F_\gamma = \eta^{\mu\nu} \partial_\mu h_{\nu\gamma} - \left(\frac{\varrho - 1}{\varrho - 2} \right) \partial_\gamma h, \quad h \equiv \eta^{\mu\nu} h_{\mu\nu}, \quad \zeta_{\alpha\beta} = 0, \quad (36)$$

where ϱ is an arbitrary parameter. According to Eq. (34), ϱ -dependence of the effective metric is given by

$$\frac{\partial h_{\mu\nu}^{\text{eff}}}{\partial \varrho} = \frac{\delta \langle \bar{c}^\alpha \partial F_\alpha / \partial \varrho \rangle}{\delta k^{\mu\nu}} \bigg|_{\substack{J \setminus j=0 \\ K=0}}. \quad (37)$$

There are two diagrams with the structure of Fig. 2(a), in which the scalar particle propagates in opposite directions. They are represented in Fig. 3. In fact, it is sufficient to evaluate either of them. Indeed, these diagrams have the following tensor structure

$$a_1 q_\mu q_\nu + a_2 (p_\mu q_\nu + p_\nu q_\mu) + a_3 p_\mu p_\nu + a_4 \eta_{\mu\nu},$$

where $a_i, i = 1, \dots, 4$, are some functions of p^2 . When transformed to the coordinate space, the second and third terms become spacetime gradients, hence, they can be written in the form $D_{\mu\nu}^\alpha \Xi_\alpha$. As was discussed in the preceding sections, the terms of this type respect general covariance, therefore, we can restrict ourselves to the calculation of a_1 and a_4 only. On the other hand, diagrams of Fig. 3 go over one into another under the substitution $q \rightarrow p - q$ which leaves a_1, a_4 unchanged.

Calculation of the diagram 3(a) is somewhat easier. Its analytical expression

$$\begin{aligned} I_{3(a)} = & -i \frac{\mu^\epsilon}{\sqrt{2\varepsilon_{\mathbf{q}} 2\varepsilon_{\mathbf{q}-\mathbf{p}}}} \int \frac{d^{4-\epsilon} k}{(2\pi)^4} \left\{ \frac{1}{2} W^{\alpha\beta\gamma\delta} (q_\gamma - p_\gamma) (k_\delta + q_\delta) - m^2 \frac{\eta^{\alpha\beta}}{2} \right\} \\ & \times G^\phi(q+k) \left\{ \frac{1}{2} W^{\rho\tau\sigma\lambda} q_\sigma (k_\lambda + q_\lambda) - m^2 \frac{\eta^{\rho\tau}}{2} \right\} G_{\rho\tau\pi\omega}(k) \frac{\partial F^{\xi,\pi\omega}}{\partial \varrho} \\ & \times \tilde{G}_\xi^\zeta(k) \left\{ -(k_\zeta + p_\zeta) \delta_{\mu\nu}^{\chi\theta} + \delta_{\zeta\mu}^{\chi\theta} k_\nu + \delta_{\zeta\nu}^{\chi\theta} k_\mu \right\} G_{\chi\theta\alpha\beta}(k+p), \end{aligned} \quad (38)$$

where the following notation is introduced:

$$W^{\alpha\beta\gamma\delta} = \eta^{\alpha\beta} \eta^{\gamma\delta} - \eta^{\alpha\gamma} \eta^{\beta\delta} - \eta^{\alpha\delta} \eta^{\beta\gamma},$$

μ – arbitrary mass scale, $\varepsilon_{\mathbf{q}} = \sqrt{\mathbf{q}^2 + m^2}$, and $\epsilon = 4 - d$, d being the dimensionality of spacetime. Explicit expressions for the propagators

$$\begin{aligned} G_{\mu\nu\sigma\lambda} = & -\frac{W_{\mu\nu\sigma\lambda}}{\square} + \varrho (\eta_{\mu\nu} \partial_\sigma \partial_\lambda + \eta_{\sigma\lambda} \partial_\mu \partial_\nu) \frac{1}{\square^2} \\ & - (\eta_{\mu\sigma} \partial_\nu \partial_\lambda + \eta_{\mu\lambda} \partial_\nu \partial_\sigma + \eta_{\nu\sigma} \partial_\mu \partial_\lambda + \eta_{\nu\lambda} \partial_\mu \partial_\sigma) \frac{1}{\square^2} \\ & - (3\varrho^2 - 4\varrho) \partial_\mu \partial_\nu \partial_\sigma \partial_\lambda \frac{1}{\square^3}, \\ \tilde{G}_\beta^\alpha = & -\frac{\delta_\beta^\alpha}{\square} + \frac{\varrho}{2} \frac{\partial^\alpha \partial_\beta}{\square^2}, \\ G^\phi = & \frac{1}{\square + m^2}, \end{aligned} \quad (39)$$

Calculation of (38) can be further simplified using the relation

$$\frac{\partial F_1}{\partial \varrho} G = -F_1 \frac{\partial G}{\partial \varrho},$$

which follows from $F_1 G = 0$, and noting that all gradient terms in the graviton propagators, contracted with the $\phi^2 h$ vertices, can be omitted (see Sec. III A), i.e., only the first line in Eq. (39) actually contributes.

Let the equality of two functions up to a spacetime diffeomorphism be denoted by “ \sim ”. Then, performing tensor multiplications in Eq. (38), and omitting terms proportional to p_μ , one obtains

$$\begin{aligned}
I_{3(a)} \sim & -i \frac{\mu^\epsilon}{\sqrt{2\varepsilon_{\mathbf{q}} 2\varepsilon_{\mathbf{q}-\mathbf{p}}}} \int \frac{d^{4-\epsilon} k}{(2\pi)^4} \frac{1}{k^4} \frac{1}{(k+p)^2} \frac{1}{m^2 - (k+q)^2} \\
& \left\{ \eta_{\mu\nu} \left[\frac{\varrho}{2} (k^4 + 2PQ)(P-Q)(Q-m^2) \right. \right. \\
& + \frac{\varrho}{2} k^2 P(P+Q)(Q-m^2) - \varrho k^2 Q^2(Q-m^2) \\
& + \left(\frac{\varrho}{4} p^2 (k^2 + 2Q) + (k+p)^2 m^2 \right) (k^2 + P)(Q-m^2) \Big] \\
& + k_\mu k_\nu \left[\varrho (P^2 + k^2 m^2)(Q-m^2) + 4\varrho PQm^2 - 3\varrho PQ^2 \right. \\
& + \frac{\varrho}{2} p^2 (P-2Q)(Q-m^2) + 2\varrho Q^2(Q-m^2) - \varrho Pm^4 \\
& + 2(k+p)^2 (Q(Q-2m^2) - P(Q-m^2) + m^4) \Big] \\
& + 2(k_\mu q_\nu + k_\nu q_\mu)(k+p)^2 (Q-P)(Q-m^2) \\
& \left. \left. - 2q_\mu q_\nu (k+p)^2 (k^2 + P)(Q-m^2) \right\}, \quad Q \equiv (kq), \quad P \equiv (kp). \tag{40}
\end{aligned}$$

Introducing the Schwinger parametrization of denominators

$$\begin{aligned}
\frac{1}{k^2} &= - \int_0^\infty dy \exp\{y k^2\}, \quad \frac{1}{(k+p)^2} = - \int_0^\infty dx \exp\{x(k+p)^2\}, \\
\frac{1}{k^2 + 2(kq)} &= - \int_0^\infty dz \exp\{z[k^2 + 2(kq)]\},
\end{aligned}$$

one evaluates the loop integrals using

$$\begin{aligned}
& \int d^d k \exp\{k^2(x+y+z) + 2k^\mu(xp_\mu + zq_\mu) + p^2 x\} \\
&= i \left(\frac{\pi}{x+y+z} \right)^{d/2} \exp\left\{ \frac{p^2 xy - m^2 z^2}{x+y+z} \right\}, \\
& \int d^d k \, k_\alpha \exp\{k^2(x+y+z) + 2k^\mu(xp_\mu + zq_\mu) + p^2 x\} = \\
&= i \left(\frac{\pi}{x+y+z} \right)^{d/2} \exp\left\{ \frac{p^2 xy - m^2 z^2}{x+y+z} \right\} \left[-\frac{xp_\alpha + zq_\alpha}{x+y+z} \right],
\end{aligned}$$

etc., up to six k -factors in the integrand. This calculation can be automated to a considerable extent with the help of the tensor package [11] for the REDUCE system. Changing the integration variables (x, y, z) to (t, u, v) via

$$x = \frac{t(1+t+u)v}{m^2(1+\alpha tu)}, \quad y = \frac{u(1+t+u)v}{m^2(1+\alpha tu)}, \quad z = \frac{(1+t+u)v}{m^2(1+\alpha tu)}, \quad \alpha \equiv -\frac{p^2}{m^2},$$

integrating v out, subtracting the ultraviolet divergence⁷

$$I_{3(a)}^{\text{div}} = -\frac{1}{32\varepsilon_{\mathbf{q}}\pi^2\epsilon} \left(\frac{\mu}{m}\right)^\epsilon \left[\frac{1}{3}q_\mu q_\nu + \eta_{\mu\nu}(p^2 - 2m^2) \frac{3\varrho - 2}{24} \right],$$

setting $\epsilon = 0$, omitting gradient terms, and retaining only the \hbar^0 -contribution, we obtain

$$\begin{aligned} I_{3(a)}^{\text{ren}} &\equiv (I_{3(a)} - I_{3(a)}^{\text{div}})_{\epsilon \rightarrow 0} \\ &\sim \frac{m^2}{32\varepsilon_{\mathbf{q}}\pi^2} \int_0^\infty \int_0^\infty du dt \left\{ \frac{\eta_{\mu\nu}\varrho}{H^3\alpha} \left(\frac{1}{D^2} - \frac{1}{2D} \right) \right. \\ &\quad \left. + \frac{q_\mu q_\nu}{H^3 m^2} \left[\frac{2H^2}{D^2} \left(1 - \frac{1}{D} \right) + \frac{1}{\alpha} \left(\frac{4\varrho}{D^3} - \frac{11\varrho + 4}{D^2} + \frac{8\varrho + 4}{D} \right) \right] \right\}, \\ D &\equiv 1 + \alpha u t, \quad H \equiv 1 + u + t. \end{aligned} \tag{41}$$

The root singularity in the right hand side of Eq. (41) can be extracted using the formulae derived in the appendix of Ref. [2]. Denoting

$$\int_0^\infty \int_0^\infty du dt H^{-n} D^{-m} \equiv J_{nm},$$

one has

$$\begin{aligned} J_{12}^{\text{root}} &= \frac{\pi^2}{4\sqrt{\alpha}}, \quad J_{13}^{\text{root}} = \frac{3\pi^2}{16\sqrt{\alpha}}, \\ J_{31}^{\text{root}} &= -\frac{\pi^2}{16}\sqrt{\alpha}, \quad J_{32}^{\text{root}} = -\frac{3\pi^2}{32}\sqrt{\alpha}, \quad J_{33}^{\text{root}} = -\frac{15\pi^2}{128}\sqrt{\alpha}. \end{aligned}$$

Substituting these into Eq. (41) gives

$$I_3^{\text{ren}} = I_{3(a)}^{\text{ren}} + I_{3(b)}^{\text{ren}} \sim 2I_{3(a)}^{\text{ren}} \sim \frac{1}{256\varepsilon_{\mathbf{q}}\sqrt{\alpha}} [q_\mu q_\nu (\varrho + 1) - \eta_{\mu\nu} m^2 \varrho]. \tag{42}$$

Finally, restoring ordinary units, and going over to the coordinate representation with the help of

$$\int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{e^{i\mathbf{p}\mathbf{x}}}{|\mathbf{p}|} = \frac{1}{2\pi^2 r^2},$$

⁷A technicality must be mentioned here. By itself, the diagram of Fig. 1 is free of infrared divergences. As a result of the BRST-operating with this diagram, however, some fictitious infrared divergences are brought into individual diagrams representing the right hand side of Eq. (24). This is because the vertex $D_{\mu\nu}^\alpha C_\alpha$ contains the term $C^\alpha \partial_\alpha h_{\mu\nu}$ in which the spacetime derivatives act on the gravitational, rather than the ghost field. These divergences occur as $u, t \rightarrow \infty$. They are proportional to integer powers of p^2 , and therefore do not interfere with the part containing the root singularity. Since these divergences must eventually cancel in the total sum in Eq. (24), they will be simply omitted in what follows.

we obtain the following expression for the ϱ -derivative of the G^2 -order contribution to the effective metric

$$\frac{\partial h_{\mu\nu}^{\text{eff}}}{\partial \varrho} = \frac{\partial h_{\mu\nu}^{\text{tree}}}{\partial \varrho} + \frac{\partial h_{\mu\nu}^{\text{loop}}}{\partial \varrho} \sim \frac{\partial h_{\mu\nu}^{\text{loop}}}{\partial \varrho} \sim \frac{G^2 m}{2\varepsilon_{\mathbf{q}} c^2 r^2} \left[\frac{q_\mu q_\nu}{c^2} (\varrho + 1) - \eta_{\mu\nu} m^2 \varrho \right]. \quad (43)$$

The right hand side of this equation cannot be represented in the form (23). This result can be made more expressive by calculating the ϱ -variation of the scalar curvature. Setting $\mathbf{q} = 0$ for simplicity, we find that a variation $\delta\varrho$ produces a non-zero variation of R :

$$\begin{aligned} \delta R[h^{\text{eff}}(x(\sigma^0))] &= \frac{\delta R}{\delta h_{\mu\nu}^{\text{eff}}} \left(\frac{\partial h_{\mu\nu}^{\text{tree}}}{\partial \varrho} + \frac{\partial h_{\mu\nu}^{\text{loop}}}{\partial \varrho} \right) \delta \varrho + \frac{\partial R}{\partial x^\mu} \delta x^\mu \\ &= \frac{\partial R}{\partial x^\alpha} \Xi_{\text{tree}}^\alpha + (\partial^\mu \partial^\nu - \eta^{\mu\nu} \square) \frac{h_{\mu\nu}^{\text{loop}}}{\partial \varrho} \delta \varrho - \frac{\partial R}{\partial x^\mu} \Xi_{\text{tree}}^\mu \\ &= \partial_i \partial_k \frac{\partial h_{ik}^{\text{loop}}}{\partial \varrho} \delta \varrho + \Delta \frac{\partial h^{\text{loop}}}{\partial \varrho} \delta \varrho = \frac{G^2 m^2}{c^4 r^4} (1 - 2\varrho) \delta \varrho, \end{aligned}$$

or⁸

$$\frac{\partial R[h^{\text{eff}}(x(\sigma^0))]}{\partial \varrho} = \frac{G^2 m^2}{c^4 r^4} (1 - 2\varrho). \quad (45)$$

Equations (43), (45) express violation of general covariance by the loop corrections.

IV. DISCUSSION AND CONCLUSIONS

The results obtained in the preceding sections answer the general questions stated in the Introduction. First of all, they establish general transformation properties of observable quantities under deformations of a reference frame, induced by variations of the gauge conditions. Specifically, it was shown in Sec. III A 2 that although variations of the weighting matrix lead to non-zero variations of the effective fields, the latter transform in such a way that the observable quantities remain unchanged. Thus, the seemingly wider freedom in the choice of gauge conditions at the quantum level introduces no ambiguity into the values of

⁸Another way to obtain this result is to introduce the sources tR and $kR_{,\alpha}c^\alpha$ for the scalar curvature and its BRST-variation, respectively, into the generating functional (11), instead of the corresponding sources for the metric. Then Eq. (37) is replaced by

$$\frac{\partial R^{\text{eff}}}{\partial \varrho} = \frac{\delta \langle \bar{c}^\alpha \partial F_\alpha / \partial \varrho \rangle}{\delta k} \bigg|_{\substack{J \setminus j=0 \\ K=0}}. \quad (44)$$

At the second order in G , the nontrivial contribution comes again from the diagram of Fig. 2(a) in which the lower left vertex is now generated by $R_{,\alpha}c^\alpha$. Thus, only the linear part of R gives rise to a non-zero contribution to the right hand side of Eq. (44). In other words, $\delta R[h^{\text{eff}}] = \delta R^{\text{eff}}$, though generally $R[h^{\text{eff}}] \neq R^{\text{eff}}$.

the physical quantities. In this sense, one can say that the physical properties of a given reference frame are determined essentially by the form of the functions F_α only, just like in the classical theory.

Unlike the classical general relativity, however, observations of a physical quantity in two reference frames defined by different sets of functions F_α give generally different results. For instance, spacetime curvature observed in a fixed reference point σ varies according to Eq. (45) under deformations of the reference frame, induced by variations of the parameter ϱ entering the gauge conditions (36). The loop contributions thus violate general covariance, depriving thereby the notion of spacetime curvature of its absolute meaning, which is recovered only in the macroscopic limit $N \rightarrow \infty$, where N is the number of elementary particles producing the given gravitational field.

Thus, we arrive at the conclusion that *the principle of general covariance is to be considered as approximate, valid only for the description of macroscopic phenomena.*

Let us now discuss this issue from the practical point of view. As was mentioned in the Introduction, from the point of view of formal power expansion in \hbar , the characteristic length scale of quantum gravity effects is the same as in the classical Einstein theory. Informally, the actual value of this scale depends on the physical properties of a system under consideration. For fundamental elementary particle such as the electron, r_g is even smaller than the Planck length. For the stars r_g is measured by kilometers, but the quantum contribution is highly suppressed in this case; in comparison with the classical (tree) contribution, the loop contribution to the first post-Newtonian correction to the field of a gravitating body contains the extra factor $1/N$, where N is the number of constituent particles. For the solar gravitational field, for instance, this factor is of the order $m_{proton}/M_\odot \approx 10^{-57}$. However, this suppression is only in force as long as interactions of the particles are relatively small. This differs in a situation when the evolution of a system of particles ends up with formation of the horizon. In this case, interaction of particles in no way can be considered small. From the point of view of an external observer, the number N is now irrelevant to the gravitational field of the collapsar (this is a consequence of the “no hair” theorem). Made by the infinite gravitational force indivisible, this object can be considered as a “particle”, *i.e.*, N is to be set unity. As is well known, black holes of certain types do behave like normal elementary particles [12]. On the other hand, it should be emphasized that from the point of view of the low-energy theory we work with, the exact structure of the microscopic theory in which black hole is embedded is of no importance. It is only important whether or not this object can be described by a single quantum field. The loop contributions to the gravitational field of black hole are thus of the same order of magnitude as the ordinary post-Newtonian corrections predicted by classical general relativity.

Calculation of the actual value of the one-loop contribution to the effective gravitational field of black holes can be found in Ref. [13]. Let us note in this connection that not only the static gravitational field of black holes, but also emission of the gravitational waves by the black hole binaries must be affected by the quantum contributions. The LIGO and VIRGO [14] gravitational wave detectors, which are currently under construction, will hopefully bring light into this issue.

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FIGURES

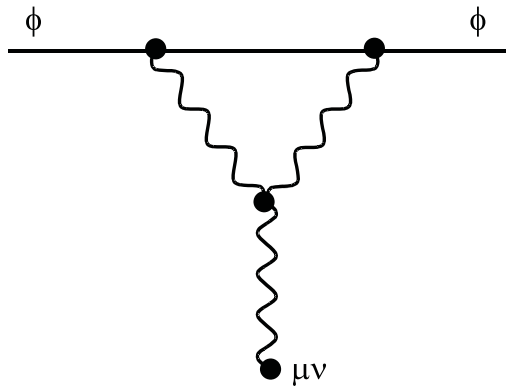


FIG. 1. The one-loop diagram contributing to the first post-Newtonian correction. Wavy lines represent gravitons, full lines massive particle.

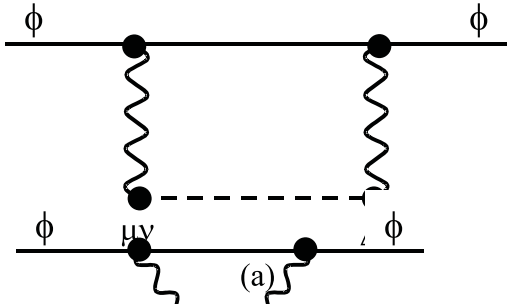


FIG. 2. The one-loop diagrams giving rise to the root singularity in the right hand side of Eq. (26). Dashed lines represent the Faddeev-Popov ghosts.

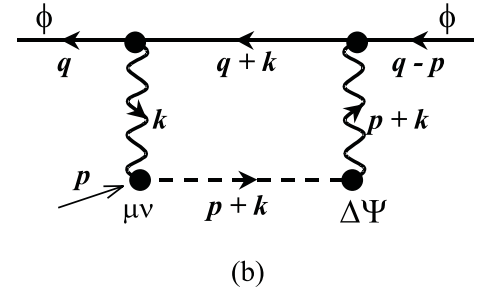
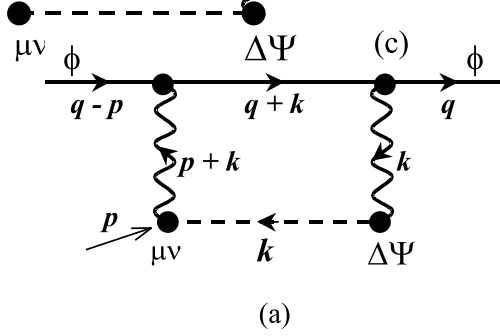


FIG. 3. Diagrams responsible for the nontrivial contribution to the right hand side of Eq. (34).